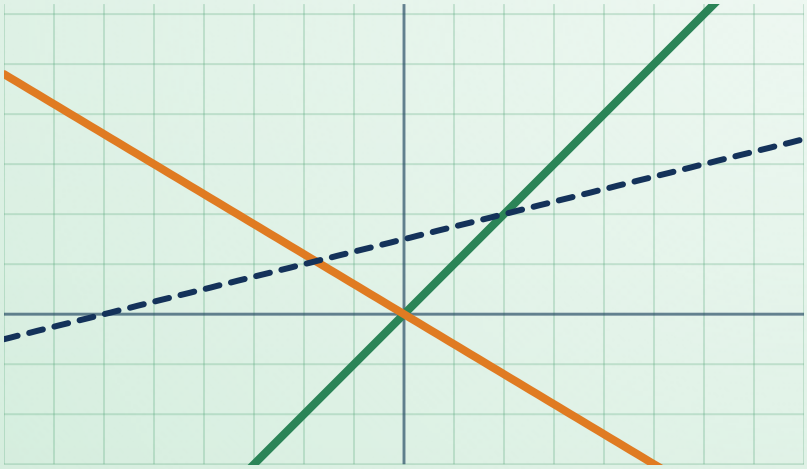
KECUDO

2026 EDITION · COLLEGE MATH SERIES



LINEAR ALGEBRA

Class E2

Vector Spaces & Linear Transformations

The structure behind every system

PREMIUM EDITION · KECUDO · E2

13 CHAPTERS · 120 SELF-ASSESSMENT QUESTIONS

INTERACTIVE EDITION + PRINT PDF

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FOUNDATION

Chapter 1

Vector Spaces & Subspaces

A **vector space** is any set of objects you can add and scale, following the usual rules (commutative, associative, a zero vector, negatives, distributive laws). Arrows in R^n, polynomials, matrices and functions are all vector spaces.

SUBSPACE TEST

A subset H of a vector space is a **subspace** if it contains 0, is closed under addition, and is closed under scalar multiplication.

WORKED EXAMPLE **Which are subspaces of R^3?**

1. The plane $x + y + z = 0$: contains 0, sums and multiples stay on it. Yes.
2. The plane $x + y + z = 1$: doesn't contain 0. No.
3. The line $t(1, 2, 3)$: yes.

✓ **Subspaces must pass through the origin**

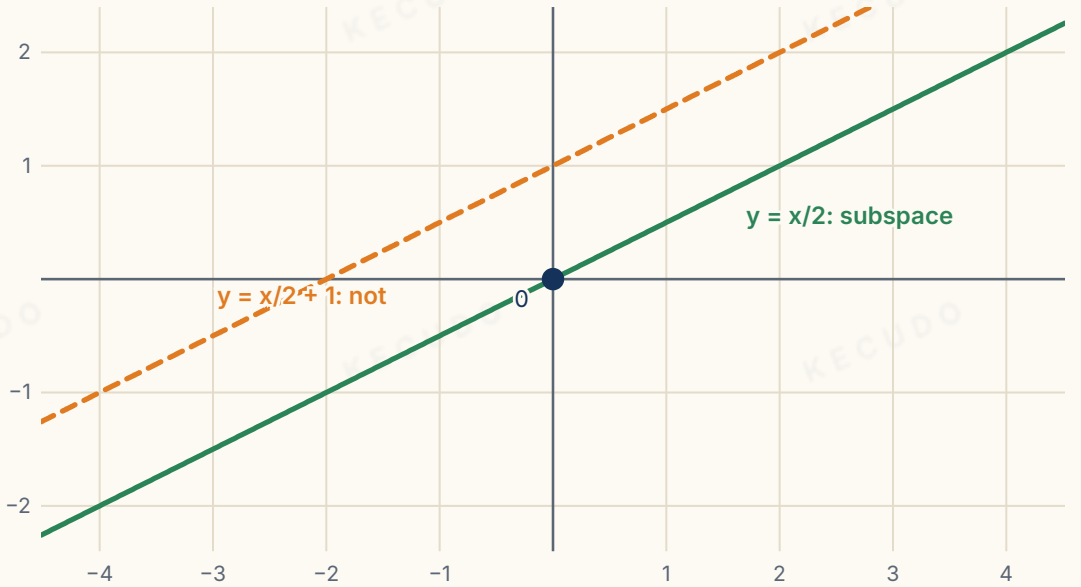


FIG 1.1 A line through the origin is a subspace of R^2; a shifted line is not.

WORKED EXAMPLE **Polynomials**

1. P_2 = all polynomials of degree at most 2: $a + bt + ct^2$.
2. Adding or scaling them gives another such polynomial.

✓ **P_2 is a vector space**

FOUNDATION

Chapter 2

Null Space & Column Space

Space	Definition	Lives in
Nul A	all x with $Ax = 0$	R^n (inputs)
Col A	all combinations of the columns	R^m (outputs)

WORKED EXAMPLE **A = [1 2 3; 2 4 6]**

1. $Ax = 0$ reduces to $x_1 + 2x_2 + 3x_3 = 0$, with x_2, x_3 free.
2. Nul A = Span$\{(-2, 1, 0), (-3, 0, 1)\}$, a plane in R^3.
3. Every column is a multiple of $(1, 2)$: Col A = Span$\{(1, 2)\}$, a line in R^2.

✓ **Null space: dimension 2. Column space: dimension 1**

WHAT THEY MEAN

Col A is the set of b for which $Ax = b$ is solvable. Nul A is the set of inputs the matrix sends to zero: the ambiguity in solutions.

Pivot columns of A itself (not of its echelon form) form a basis for Col A. The vectors from the parametric solution of $Ax = 0$ form a basis for Nul A.

CHECK YOUR UNDERSTANDING

Chapter 1: Vector Spaces & Subspaces

5 questions · your answer locks when you choose it

1. Every subspace must contain:

- **A** the zero vector
- **B** only positive vectors
- **C** $(1, 1, 1)$
- **D** a basis of R^n

2. Is the line $y = 2x + 1$ a subspace of R^2?

- **A** No
- **B** Can't tell
- **C** Yes
- **D** Only for $x > 0$

3. Is the set of all 2×2 matrices a vector space?

- **A** Only invertible ones
- **B** No
- **C** Yes
- **D** Only symmetric ones

4. Is $\text{Span}\{v_1, v_2\}$ always a subspace?

- **A** Only in R^2
- **B** Only if independent
- **C** Yes
- **D** No

5. The first quadrant $x, y \geq 0$ is:

- **A** all of R^2
- **B** a subspace
- **C** not a subspace (not closed under -1)
- **D** a basis

FINAL EXAM

25 Questions on the Whole Book

Parts A–E · 5 questions each

Part A · Basics

1. A subspace must contain:

A the zero vector
B the identity
C all of R^n
D a nonzero vector

2. A basis is:

A any spanning set
B any independent set
C the zero vector
D an independent spanning set

3. The dimension of a space is:

A always 3
B the number of vectors in a basis
C its rank
D the number of vectors in it

4. rank A equals:

A dim Nul A
B the number of rows
C the number of zeros
D the number of pivots

5. A linear transformation preserves:

A sums and scalar multiples
B nothing
C angles always
D lengths always

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Thank you for learning with Kecudo.

NEXT IN THE SERIES

Class E3 · Determinants, Eigenvalues & Eigenvectors